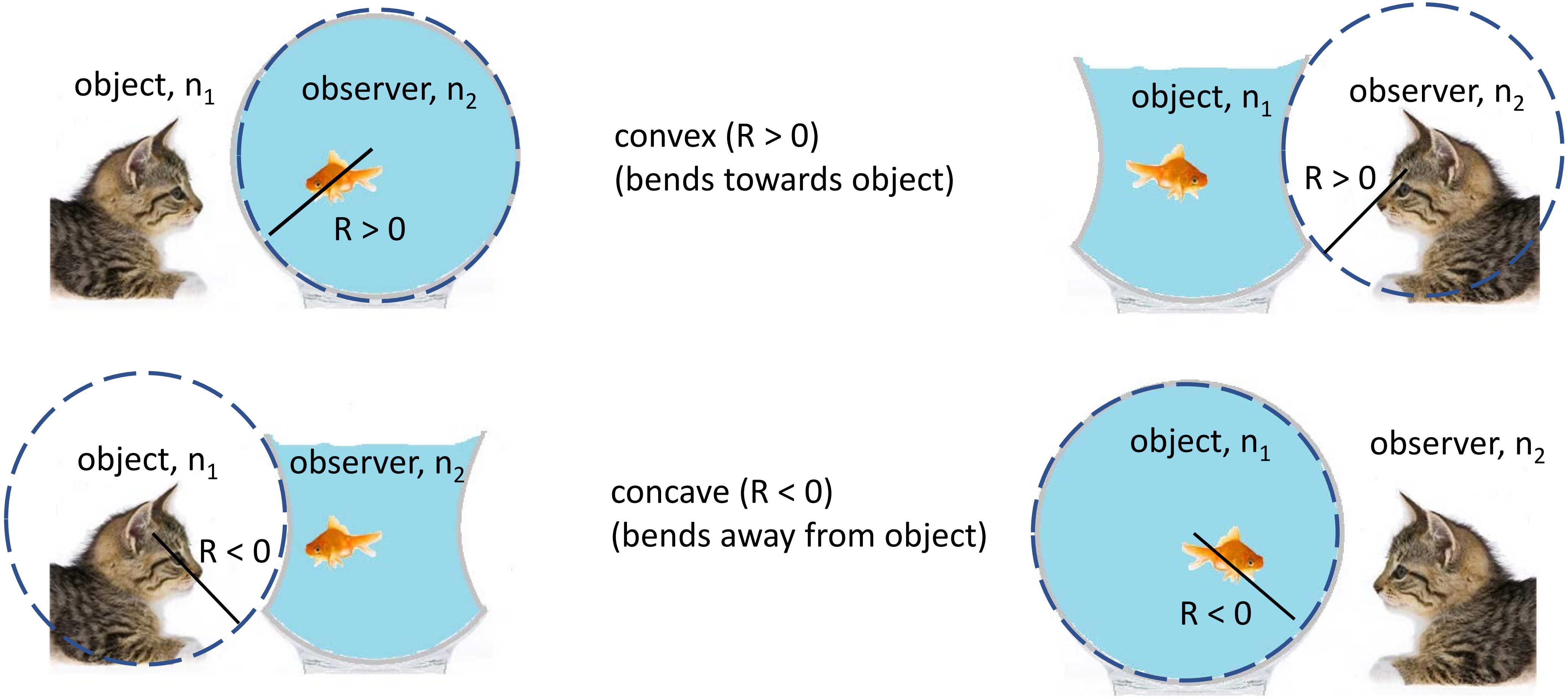
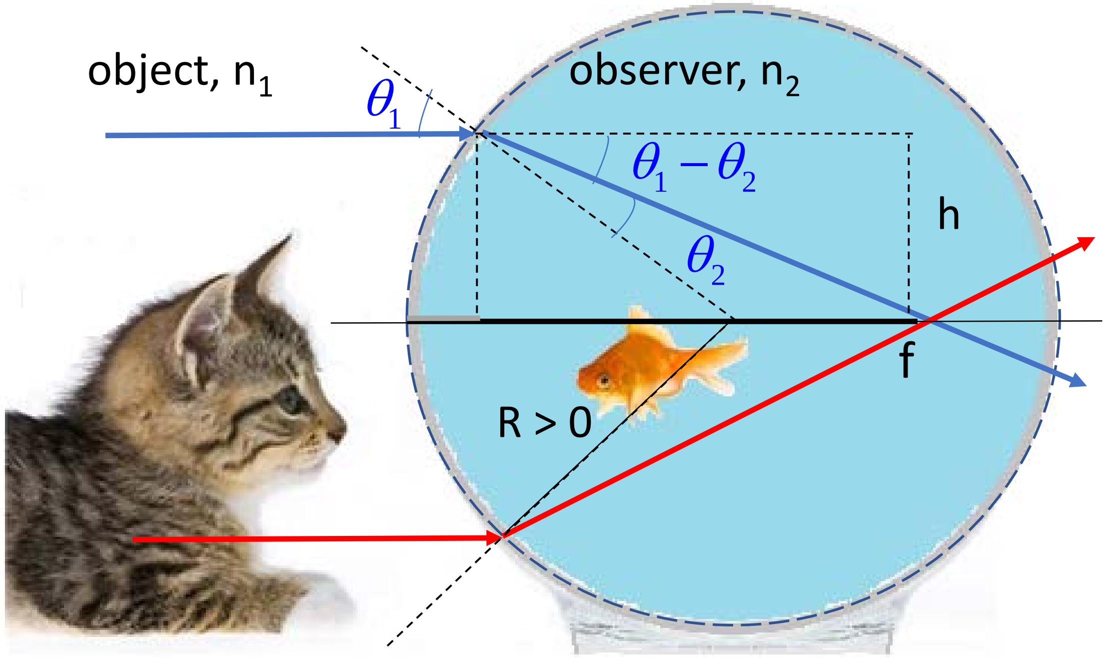
E.3 Refracting Surfaces

Now we want to examine the images formed by refracting surfaces. This scenario occurs when you have the object in a medium with index of refraction n_1 and the observer in a medium with a different index of refraction n_2. Such a situation prevails below. On the left, the cat is the object of the observer fish, and on the right, the fish is the object of the observer cat. Like with mirrors, we can have a concave or convex surface, and each with an associated radius of curvature.



E.3 Refracting Surfaces

Like with mirrors, the first order of business is to work out the focal length of these surfaces. Remember the focal point is the point where parallel light rays will converge.



'Focusing' on *this* arrangement, which has same result as the others….

$$\frac{h}{f} \approx \tan(\theta_1-\theta_2) \qquad \text{(ignoring grey segment)}$$

$$\approx \theta_1-\theta_2 \qquad \text{(small angle approximation)}$$

$$\approx \sin\theta_1-\sin\theta_2 \qquad \text{(small angle approximation)}$$

$$=\sin\theta_1-\frac{n_1}{n_2}\sin\theta_1 \qquad \text{(Snell's law)}$$

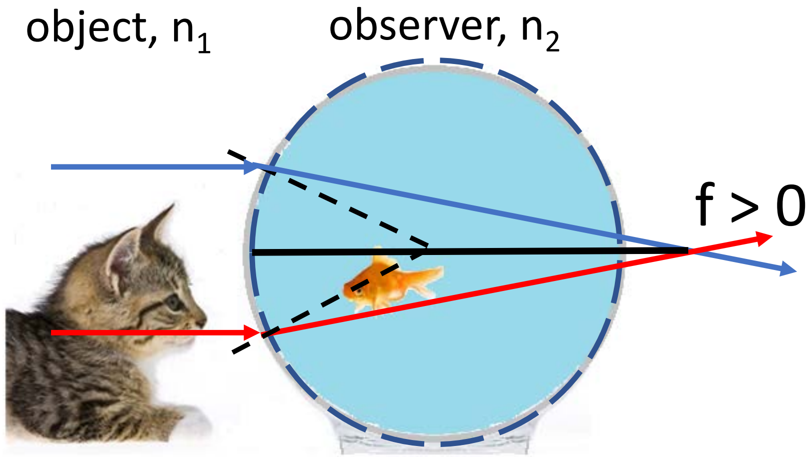
$$=\frac{h}{R}-\frac{n_1}{n_2}\frac{h}{R}$$

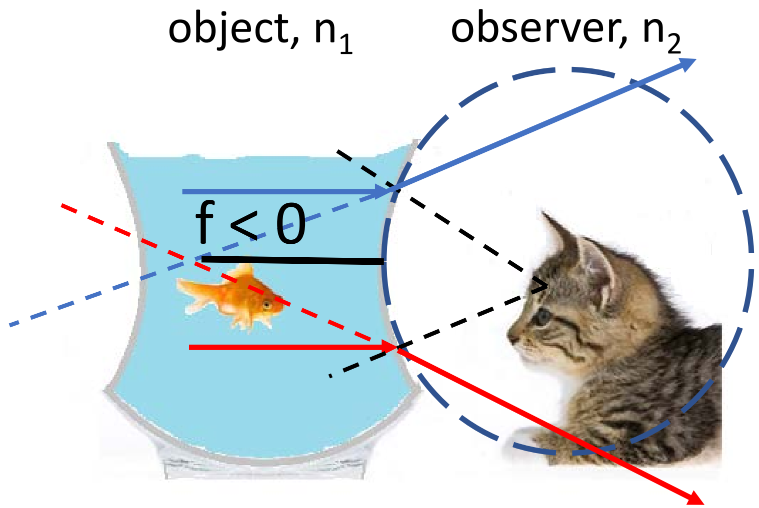
$$=\left(1-\frac{n_1}{n_2}\right)\frac{h}{R}$$

$$f \approx \frac{R}{1-n_1/n_2}$$

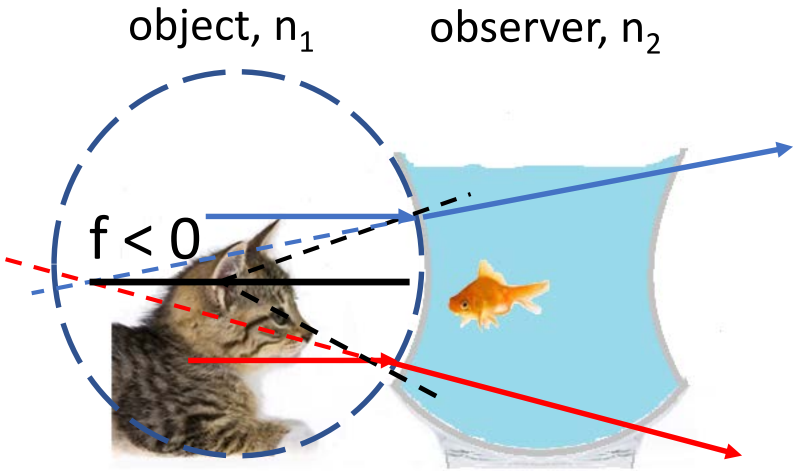
E.3 Refracting Surfaces

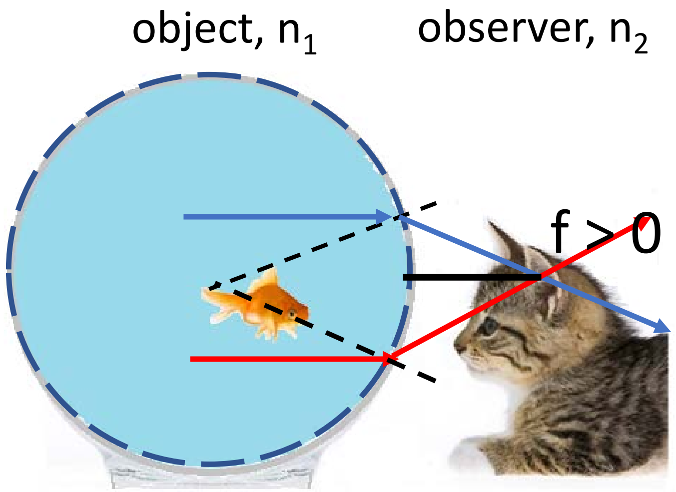
The focal length can be negative, as you might observe from the preceding equation. It's useful to illustrate what that looks like physically….



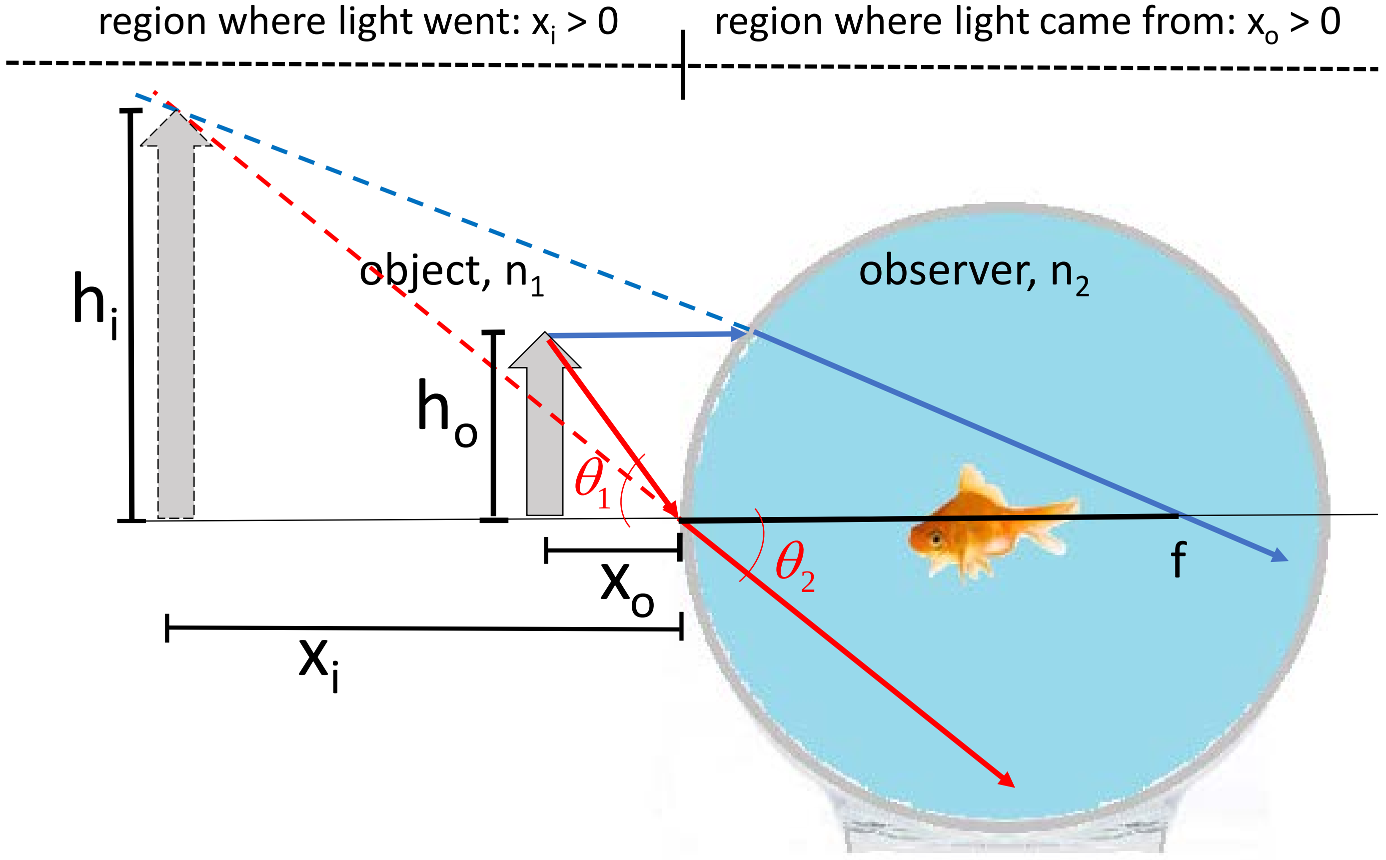


$$f \approx \frac{R}{1 - n_1 / n_2}$$





E.3 Refracting Surfaces



With a formula for the focal length in hand, we can now work out a formula for the location of an image. We will liberally use the small angle approximation…

Blue transmission

$$y_{blue} = y_0 + slope \cdot x = h_0 - \frac{h_o}{f} x$$

Red transmission

$$\begin{aligned} y_{red} &= y_0 + slope \cdot x = 0 - \tan\theta_2 \cdot x \\ &\approx -\sin\theta_2 \cdot x = -\frac{n_1}{n_2}\sin\theta_1 \cdot x \\ &\approx -\frac{n_1}{n_2}\tan\theta_1 \cdot x = -\frac{n_1}{n_2}\frac{h_o}{x_o} \cdot x \end{aligned}$$

Intersection:

$$y_{blue} = y_{orange} \longrightarrow \cancel{h_0} - \frac{\cancel{h_o}}{f} x_i = -\frac{n_1}{n_2}\frac{\cancel{h_o}}{x_o} \cdot x_i$$

$$1 - \frac{1}{f} x_i = -\frac{n_1}{n_2}\frac{1}{x_o} x_i$$

$$\boxed{\frac{n_1}{n_2}\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f}}$$

Height of image is y-coordinate of the intersection:

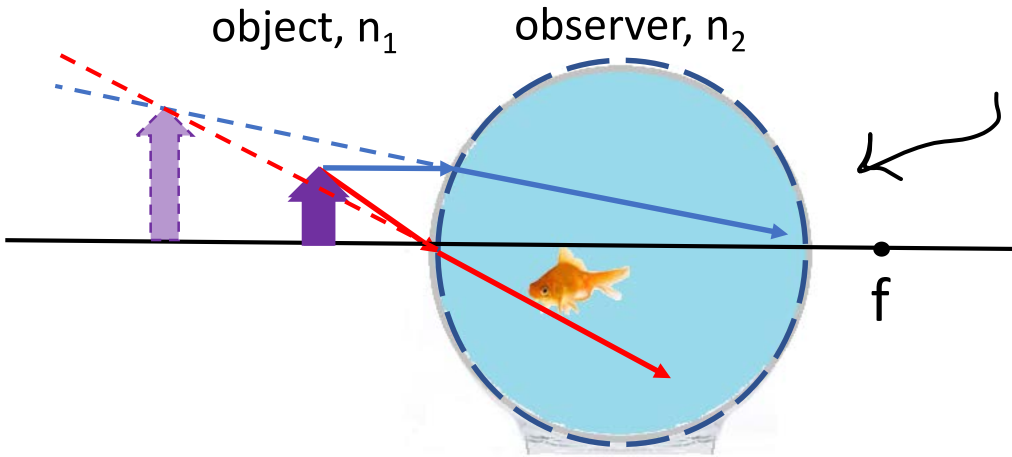
$$h_i = -\frac{n_1}{n_2}\frac{h_o}{x_o} x_i$$

So magnification is:

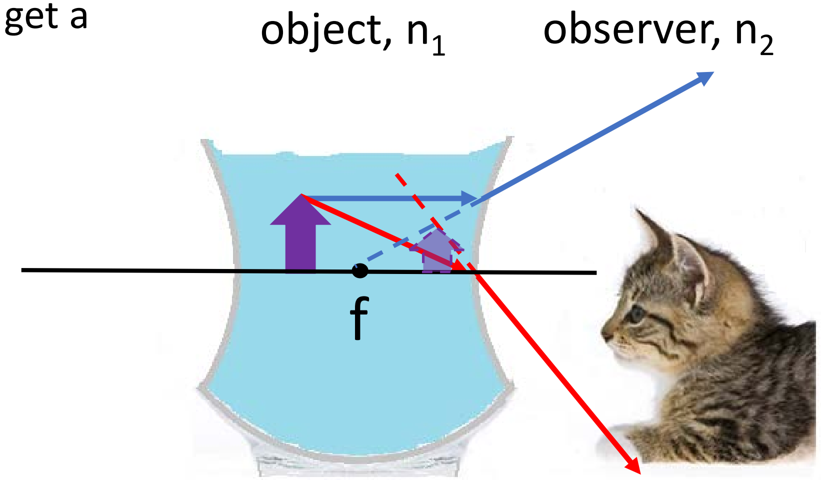
$$\boxed{m \equiv \frac{h_i}{h_o} = -\frac{n_1}{n_2}\frac{x_i}{x_o}}$$

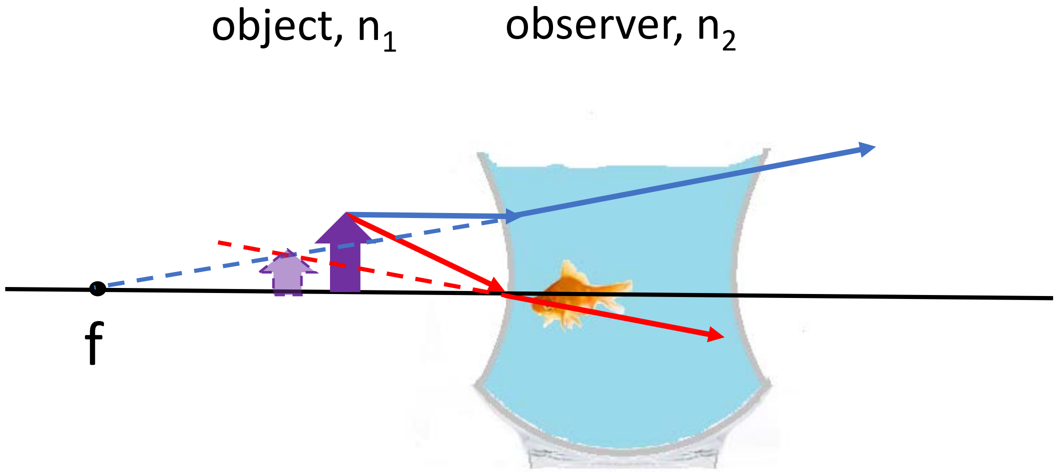
E.3 Refracting Surfaces

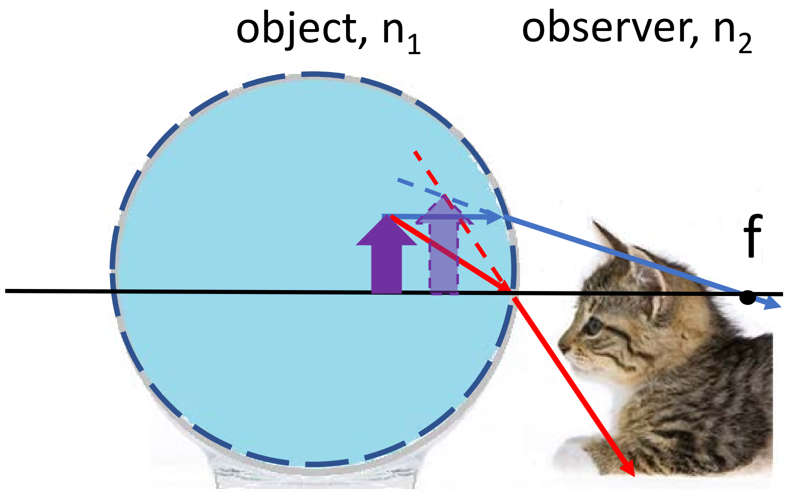
Let's see, roughly, how we would locate an image in each of the four cases…



could conceivably get a real image inside fluid if its f were small enough.

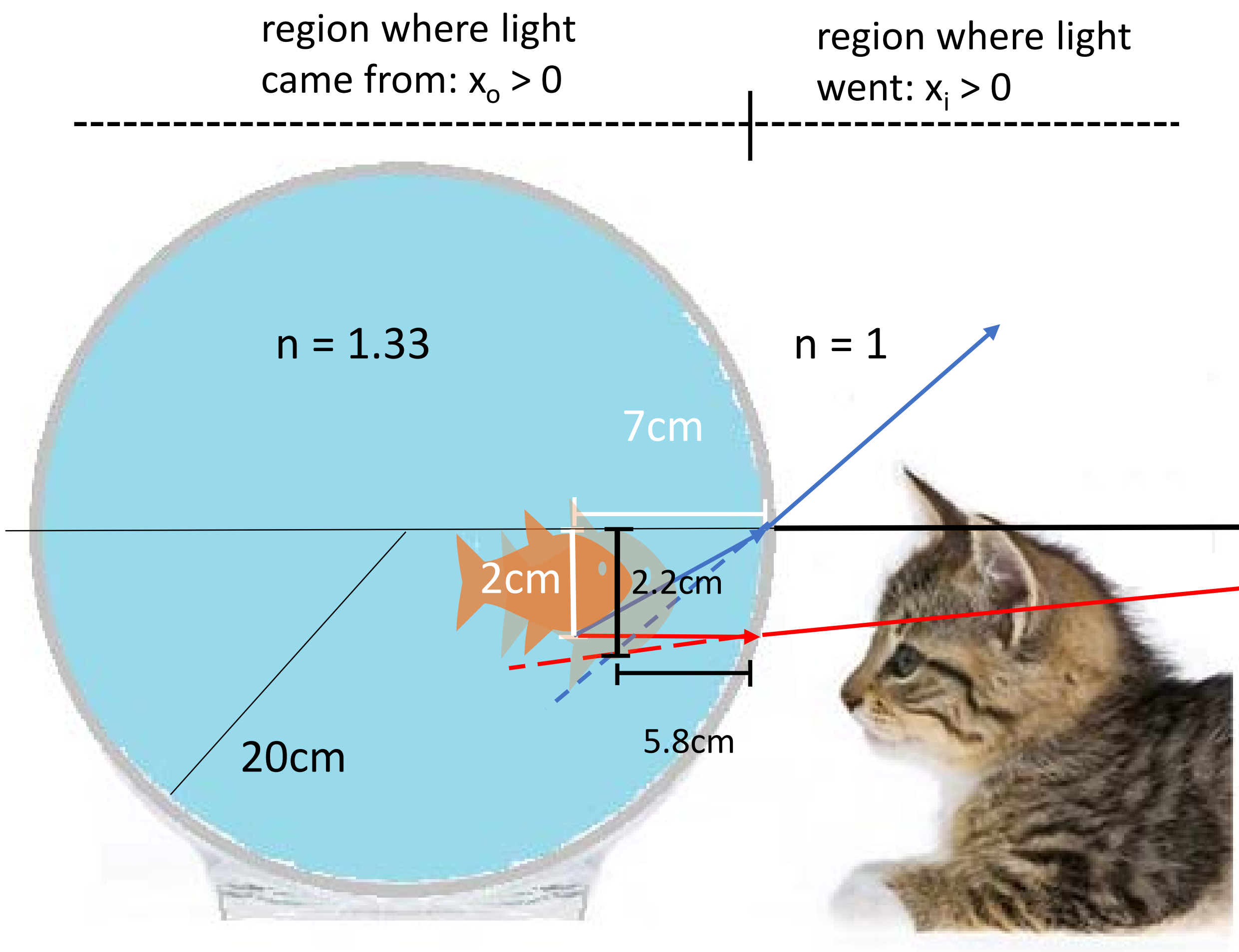






E.3 Refracting Surfaces

Say our 2cm tall fish is 7cm away from the edge of the fish bowl. Where and how tall does its fish face appear to be to cat? Do both a calculation, and drawing of the image using the ray tracing stuff.



First gotta calculate the focal length:

$$f = \frac{R}{1-n_1/n_2} = \frac{-20\,\text{cm}}{1-1.33/1} = 60\text{cm}$$

Then we use the refracting surface eqn.

$$\frac{n_1}{n_2}\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f} \longrightarrow \frac{1.33}{1}\frac{1}{7\text{cm}} + \frac{1}{x_i} = \frac{1}{60\text{cm}}$$

$$\longrightarrow x_i = \left(\frac{1}{60\text{cm}} - \frac{1.33}{1}\frac{1}{7\text{cm}}\right)^{-1}$$

$$\longrightarrow x_i = -5.8\,\text{cm}$$

Then to get height, use the magnification equation.

$$h_i = mh_o = \left(-\frac{n_1}{n_2}\frac{x_i}{x_o}\right)h_o = \left(-\frac{1.33}{1}\frac{-5.7\text{cm}}{7\text{cm}}\right)(-2\text{cm})$$

$$= -2.2\,\text{cm}$$